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Dynamical Effect of Quantum Hair

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ABSTRACT

We show that quantum hair can alter the relation between the temperature and the mass of a black hole. A Z_N electric charge on a black hole generates an electric field that is nonperturbative in $\hbar$. A Z_N magnetic charge on a black hole can be described classically, and can support a stable remnant. For global quantum hair, in contrast to gauge hair, we find no dynamical effects.

Introduction: There is considerable conceptual tension between two fundamental aspects of black hole physics. On the one hand, powerful "no–hair" theorems[1] indicate that black holes have essentially no structure; on the other hand, their response to external perturbations has a dissipative character.[2] The existing calculations of black hole entropy,[3] while giving a fairly convincing derivation of its value, do little to elucidate the nature of the microscopic states that this entropy presumably counts (entropy = ln(accessible states)). Thus, for example, the extreme Reissner-Nordström black holes have zero temperature but finite entropy $S = \pi(M/M_{\text{Planck}})^2$. When properly computed, scattering of particles from such a hole should allow internal degrees of freedom of the hole to be excited; it should be described by an S-matrix that keeps track of the initial and final state of the hole.

Related to the mystery of black hole entropy is another tension, between the concepts used in the theoretical description of elementary particles on the one hand and black holes on the other. In the former case, we may specify many internal quantum numbers; in the latter we are constrained by the no–hair theorems. In the former case, time evolution is realized by a unitary matrix, and we regard temperature as a derived concept; whereas in the latter case it sometimes seems (and has been seriously suggested[4]) that thermodynamics is more fundamental than quantum mechanics. Yet surely it must be that sufficiently heavy elementary particles ($M \gg M_{\text{Planck}}$) are at the same time black holes, since their uncertainty in position is negligible compared to the event horizon that they create (Compton radius $\ll$ Schwarzschild radius).

Recently, there have been several attempts to address these issues.[5–7] Our discussion here will be close to the spirit of Ref. 5, and will also elucidate the relation of Ref. 5 to Ref. 6.

A major conclusion of Ref. 5 was that black holes can carry a purely quantum-mechanical type of hair that is totally invisible classically. The arguments for the existence of such "quantum hair" have since been formalized and generalized.[8,9] But until now, no concrete effects of quantum hair on the dynamical properties of the hole have been explicitly discussed. Our aim here is to show that such effects do exist, and can substantially alter the properties of the hole—specifically, the relation between its temperature and its mass. A preliminary account of this work appeared in Ref. 10. A more detailed account will appear in Ref. 11.

We will use units with $G = c = 1$, but factors of $\hbar$ will be explicitly indicated, so that quantum effects can be easily recognized.

Virtual string loops: For simplicity and concreteness, we will focus on the simplest case of quantum hair, that associated with a Z_N gauge symmetry.[5] This discrete local symmetry arises in a $U(1)$ gauge theory with fundamental charge unit $\hbar e$, if bosons with charge $N\hbar e$ condense. The residual charge that cannot be fully screened by the condensate is defined modulo N. The charge of an object can be measured at long range by observing the Aharonov- Bohm interference pattern generated when the object passes on either side of a cosmic string that carries magnetic flux $2\pi/Ne$.

Even in the absence of actual strings, there is a dynamical mechanism by which a Z_N charge affects the intrinsic properties of a particle that carries it—the charge modulates the phase of the amplitude for virtual processes where a string world sheet envelops the particle. For a charge Q and a string of minimal flux, this phase is $\exp(i2\pi Q/N\hbar e)$.

Here we are particularly interested in the case where the particle is a black hole. There is one limit in which the leading effect of Z_N charge on a black hole can easily be estimated heuristically. That is when the intrinsic thickness μ^{-1} of the string is small compared to the hole radius—the "thin-string" limit. (Here μ^{-1} is the Compton wavelength of the vector meson.) In that case, there are two major simplifications: the tidal forces on the string are too weak to disturb its internal structure, and the tension of the virtual string world sheet causes it to hug closely to the minimal surface, *i.e.* the event horizon. (In addition, the back reaction of the string on the background geometry can be safely neglected if the tension is small in Planck units.) Thus, the most important charge-dependent correction to the partition function for the black hole is due to a single world sheet wrapped closely around the event horizon, with either orientation; this correction has the form

$$Z_Q(\beta) \simeq \tilde{Z}(\beta) \left[1 + C(\beta\hbar)\cos(2\pi Q/N\hbar e)e^{-A_{\rm bh}T_{\rm string}/\hbar}\right] . \qquad (1)$$

Here $\tilde{Z}$ is the partition function when the effects of virtual strings are neglected, $A_{\rm bh}$ is the area of the event horizon, $T_{\rm string}$ is the string tension, and $C(\beta\hbar) > 0$ is a numerical factor that arises from the small fluctuations about the minimal world sheet configuration. (It could be calculated using the formalism described below.) By standard thermodynamic manipulations, this yields a correction to the expression for the inverse temperature β of a Schwarzschild black hole with mass M

$$\beta\hbar \simeq 8\pi M \left[1 - 4T_{\rm string}\; C(8\pi M)\; \cos(2\pi Q/N\hbar e)e^{-16\pi M^2 T_{\rm string}/\hbar}\right] . \qquad (2)$$

Thus, the accumulated Z_N charge lowers the temperature of a black hole of given mass, just as electric charge would have if the $U(1)$ gauge symmetry had remained unbroken. We note that the dynamical effect of Z_N charge is nonperturbative in $\hbar$; it does not show up in any finite order of the loop expansion.

To justify this heuristic treatment and to treat more general cases, one needs some formal machinery. Nonperturbative effects are most conveniently analyzed using Euclidean path integral methods. Hence, following Gibbons and Hawking,[12] we will compute the partition function for a black hole by integrating over Euclidean (imaginary time) geometries that have the topology $S^2 \times R^2$ and that approach flat space at spatial infinity. Before proceeding to an analysis of the dynamical effect of Z_N charge, it will be useful to consider, in this path integral language, the dynamical effect of ordinary electric charge on the Reissner–Nordström black hole.

Classical hair: To find the partition function for a black hole with a specified value of the electric charge, we must insert into the path integral a projection onto states of definite charge. The partition function for a black hole of charge Q thus takes the form[11]

$$Z_Q(\beta) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \exp\left(- i\frac{\omega Q}{\hbar e}\right) Z_\omega(\beta) \ , \tag{3}$$

where Z_ω can be expressed as a path integral over configurations that are periodic with period $\beta\hbar$ in imaginary time τ and that obey the constraint

$$e\int_0^{\beta\hbar} d\tau \ A_\tau(\tau, r) \underset{r=\infty}{\longrightarrow} \omega \ . \tag{4}$$

In the limit $\hbar \to 0$, with $\beta\hbar$ and Q/e held fixed, both the path integral and the ω integral can be evaluated in the steepest-descent approximation. (In this limit, the thermal wavelength becomes arbitrarily large compared to the Planck length, and the classical charge Q becomes arbitrarily large compared to the charge quantum $\hbar e$.) The ω-contour is deformed so that it passes through a saddle point on the imaginary axis. (The contour is *not* rotated; its ends must be fixed, or Z_ω will blow up.) The quantity $\omega/\beta\hbar e$ may be interpreted as the electrostatic potential difference between the black hole horizon and $r = \infty$, and the value of ω at the saddle point turns out to correspond to the potential difference due to an imaginary charge iQ residing on the hole. The saddle point of the path integral with ω now fixed at this value is nothing but the analytic continuation of the charge–Q Reissner–Nordström solution to imaginary time. Thus, the action of the Euclidean Reissner–Nordström solution gives the value of ($\beta\hbar$ times) the *grand* thermodynamic potential $\Omega(\beta, \Phi)$ of a charged black hole, evaluated in the saddle-point approximation, with the electrostatic potential energy $\Phi = i\omega/\beta$ playing the role of the chemical potential coupled to the dimensionless charge $Q/\hbar e$.[12]

If, on the other hand, we wished to compute the partition function of a black hole with specified *magnetic* charge, we could implement the charge projection by simply constraining the configurations to have a definite magnetic flux on the sphere at $r = \infty$. The saddle point in this charge sector is the analytic continuation of the magnetically charged Reissner–Nordström black hole to imaginary time, which has *real* magnetic charge. The action of this solution is the *Helmholtz* free energy $F(\beta, Q)$ of the magnetically charged hole (times $\beta\hbar$). (These two Euclidean solutions have different action because the electromagnetic contributions to their actions have opposite sign.)

Z_N electric hair: In a similar way, we may find the partition function of a black hole with a specified value of a Z_N charge by inserting a charge projection into

the Euclidean path integral. The implementation of this projection follows that described above, except that the integral over ω is replaced by a sum over $\omega = 2\pi k/N$, where k is an integer; we have[11]

$$Z_Q(\beta) = (1/N) \sum_{k=-\infty}^{\infty} \exp\Big(-i\frac{2\pi k Q}{N\hbar e}\Big) Z_k(\beta) \ . \tag{5}$$

Here Z_k is a path integral over configurations that are periodic with period $\beta\hbar$ in imaginary time τ and subject to the constraint

$$\int_0^{\beta\hbar} d\tau \ A_\tau(\tau, r) \underset{r=\infty}{\longrightarrow} \frac{2\pi}{Ne} k \ . \tag{6}$$

Eq. (6) has a remarkable interpretation. If we regard $F_{r\tau}$ as a *magnetic* field, then $\int_0^\beta d\tau \ A_\tau$ is the magnetic flux in the $r - \tau$ plane, and k is the *vorticity*, the value of the flux in units of the flux quantum $2\pi/Ne$. This integer k may be regarded as the net number of virtual string world sheets that wrap around the black hole, weighted by flux and orientation. The phase that weights the contribution with vorticity k in eq. (5) is precisely the Aharonov-Bohm phase acquired by this virtual string, if the black hole carries Z_N charge Q.

Since the charge quantum $\hbar e$ vanishes in the $\hbar \to 0$ limit, the semiclassical evaluation of $Z_Q(\beta)$ proceeds differently in the case of quantum hair than in the case of classical hair. We take the $\hbar \to 0$ limit with $Q/\hbar e$ (rather than Q/e) held fixed. And since the flux quantum is formally independent of $\hbar$, the action of a vortex becomes large compared to $\hbar$ in this limit. (Strings are classical objects.) Thus, the Q–dependence of the partition function in eq. (5) is dominated by the sectors with $k = \pm 1$.

In the thin–string limit described above, the intrinsic size of a vortex is small compared to the characteristic scale $\beta\hbar$ of the background geometry, and the saddle point of the path integral in the $k = \pm 1$ sectors is well–approximated by a point vortex at the origin of the $r - \tau$ plane. The two-sphere that sits above the origin of the plane is the Euclidean vestige of the horizon. Thus the vortex, which is independent of the coordinates θ and ϕ that parameterize the two-sphere, is a precise realization of our intuitive notion of a string world sheet that tightly envelops the hole. Evaluation of the path integral in the saddle–point approximation yields eq. (1– 2), where $C(\beta\hbar)$ is a functional determinant that is computable in principle.

If the vortex represents a virtual string world sheet, however, it may seem surprising that it has nonvanishing $F_{r\tau}$ and vanishing $F_{r\theta}$ and $F_{r\phi}$. To better understand this feature, consider a virtual process in which a loop of cosmic string grows from zero at some point on the event–horizon sphere, sweeps around the sphere, and shrinks to zero at the antipodal point. The string has magnetic field in its core; hence its motion creates an electric field orthogonal to the magnetic field and the direction of motion, an electric field in the radial direction. The time–integrated value of this radial field is independent of the velocity with which the string moves. (Although the magnitude of the electric field is proportional to the velocity, the time the string spends at any point on the sphere is inversely proportional to the velocity.)

Now we must average over the point on the sphere where the loop begins. This averaging causes the magnetic field of the string to cancel, but has no effect on the radial electric field. Our rotationally–invariant vortex represents this averaged string world sheet.

We see that, although a black hole with Z_N electric charge has no *classical* hair, a nonvanishing electric field is generated by quantum effects. The vortex and anti–vortex have electric fields of opposite sign. Thus, by summing the $k = \pm 1$ contributions to the path integral, we find that $F_{r\tau}$ acquires an expectation value proportional to $\sin(2\pi Q/N\hbar e)$. This electric field is nonperturbative in $\hbar$, and decays exponentially far from the black hole.

The action of the vortex can also be computed analytically in the "thick–string" limit $\mu \to 0$. For $\mu^{-1} >> \beta\hbar$, the Higgs field makes a negligible contribution to the action, and the vortex is well–approximated by the corresponding solution for electrodynamics coupled to gravity. We then find[11]

$$Z_Q(\beta) \simeq \tilde{Z}(\beta)\left[1 + C'(\beta\hbar)\cos(2\pi Q/N\hbar e)e^{-2\pi^2/\hbar(Ne)^2}\right] . \tag{7}$$

(In eq. (7), we have made the additional approximation of neglecting the back reaction of the vortex on the geometry. This is valid if $\beta\hbar$ is large, in Planck units, compared to $1/\hbar^{1/2}Ne$; the corrections can be computed explicitly.[11]) It is instructive to compare eq. (7) with eq. (1). We may estimate the string tension as $T_{\rm string} \sim \pi v^2$, where v is the expectation value of the Higgs field. (This estimate is correct in the Bogomol'nyi[13] limit, where the vector meson and Higgs scalar have equal masses.) Thus, our thin–string and thick–string expressions for $Z_Q(\beta)$ match when the event horizon has radius $R \sim (Nev)^{-1} = \mu^{-1}$, as one might expect.

The interpretation of eq. (7) is actually quite simple. The Z_N charge on a black hole exerts a dynamical effect by modifying the phase of the amplitude for virtual processes in which a string winds around the event horizon of the hole. Because the string has finite thickness $a \sim (Nev)^{-1}$, the action of a string world sheet has a minimum value. In flat space, this minimum is of order $4\pi a^2 T_{\rm string} \sim 4\pi^2/(Ne)^2$, in reasonable agreement with the exponential suppression found in eq. (7).

There is a troubling discontinuity in the classical description of black holes in gauge theories, when the limit of vanishing vector meson mass is considered. When the vector meson mass is precisely zero, the black hole may carry hair associated with its electric charge. But for any nonzero mass—however small—no such classical hair is allowed on a stationary black hole. An elementary particle *may* carry electric charge even for nonzero vector mass; the electric field merely decays at large distance from the particle. For a stationary black hole, however, only an electric field that is strictly zero is compatible with nonsingular behavior at the event horizon.[14]

One might hope that black hole physics will behave more smoothly in the $\mu \to 0$ limit when the effects of *quantum* hair are taken into account. Indeed, the vortex solutions that dominate the path integral for a black hole with quantum hair go smoothly to the corresponding solutions (with the same value of $\omega = 2\pi k/N$) for a black hole coupled to a massless photon. (We note that the vortex solutions are *not* static, for the phase of the condensate is τ-dependent.) Furthermore, if we also allow N to become large, then the sum in the expression eq. (5) for the partition function of a black hole with Z_N charge appears to approach the integral in the expression eq. (3) for the partition function of a black hole with ordinary electric charge.

However, as we have seen, this hope that the thermodynamic behavior of a black hole with Z_N quantum hair can smoothly approach the behavior of a black hole with classical hair is *not* realized in the model that we have considered here, at least not within the domain of validity of semiclassical methods. In part, the problem is that charge on a black hole with Z_N hair must obey $|Q| \leq N\hbar e/2$, for a larger charge would be screened by the condensate. This charge is infinitesimal compared to any fixed classical charge in the small $\hbar$ limit. (And because the charge is of order $\hbar$, we should expect it to be overlooked by a purely classical analysis of black hole hair.) We could attempt to scale $\hbar$ and N together to avoid this problem, but we then encounter another obstacle. The discreteness of vorticity can be neglected only if the action of a vortex is small compared to $\hbar$. But this requires $\hbar(Ne)^2$, the expansion parameter for Higgs loops, to be large. Thus, the sum over k cannot be replaced by an integral within the domain of validity of the saddle–point approximation.

In principle, this latter difficulty could be avoided if the symmetry breakdown were driven by a *composite* field in a theory such that all elementary quanta have charge e.

Z_N magnetic hair: There is another interesting type of Z_N charge that can reside on a black hole—Z_N magnetic charge. An $SU(N)/Z_N$ gauge theory (such that all fields transform trivially under the center Z_N of the gauge group) admits magnetic monopoles with Z_N charges. Such monopoles will arise as topological solitons if $SU(N)/Z_N$ is embedded in a simply connected gauge group that is spontaneously broken. Thus we may contemplate black holes that carry such magnetic charges.

Due to confinement, however, the magnetic field of a monopole is screened at long range. This screening is reminiscent of the screening of electric fields that can arise as a consequence of the Higgs mechanism, yet there are important differences. In particular, the screening of magnetic fields in a confining gauge theory is a *quantum* effect that is nonperturbative in $\hbar$, while the screening of electric fields in a Higgs model is a property of the solutions to the *classical* field equations. Hence, electric screening enters into the classification of black hole solutions, and prevents stationary black holes from carrying classical electric hair. But since magnetic screening does not occur in the classical approximation, it does *not* prevent stationary black holes from carrying classical magnetic hair. In spite of the screening, then, Z_N magnetic charge on a black hole endows it with a *classical* variety of hair, and an observer close to the event horizon measures a non–vanishing magnetic field.

If the screening length is much smaller than the size of the black hole, then the classical approximation is badly misleading. To analyze this case, we should first note that, although the long range magnetic field of a Z_N magnetic charge is screened, the charge can nevertheless be detected at infinite range by means of the Aharonov–Bohm interaction between monopoles and electric flux tubes.[8] Guided by electric–magnetic duality, we may write down an expression for the partition function of a black hole with definite magnetic charge that is analogous to eq. (5), but where Z_k denotes a path integral over configurations with a specified value of the *electric* flux in the $r - \tau$ plane. (The electric–magnetic duality invoked here is a variant of that described by 't Hooft[15] in a somewhat different topological context.) The limit in which the confinement length scale $\hbar/\Lambda$ is much less than $\beta\hbar$ is the "thin–string" limit; Z_k is dominated by a point electric vortex sitting at the origin. Of course, the physics responsible for the confinement of electric flux is nonperturbative. Hence, this analysis, unlike the analysis of Z_N electric hair in the thin– string limit, is not strictly semiclassical.

Alternatively, since magnetic charge is topological, we may perform the charge projection by restricting the path integral to the appropriate topological sector. In the "thick–string" (large $\hbar/\Lambda$) limit, the dominant configuration in each sector approaches a magnetically charged Reissner–Nordström solution, whose action can be reliably computed. (The nonperturbative physics at distance scale $\hbar/\Lambda$ makes a negligible contribution.) The usual semiclassical theory of black hole radiance applies in this limit.

If we consider, instead of a confining theory, a model in which $SU(N)/Z_N$ breaks to $U(1)^{N-1}$ at a small mass scale v, the magnetic field is no longer screened, and the Z_N magnetic hair is converted to ordinary magnetic hair. The thermodynamic effects of Z_N hair and ordinary hair match up smoothly. In this case, we find the same physics in the $v \to 0$ limit of the Coulomb phase as in the $\Lambda \to 0$ limit of the confinement phase.

Why is the effect of Z_N magnetic charge on black hole thermodynamics in the thick–string limit so different from the corresponding effect of Z_N electric charge? The electric charge of an elementary particle—or of a black hole with Z_N quantum hair—is of order $\hbar$, and so is irrelevant in the classical limit. In contrast, magnetic monopoles are classical objects, and the magnetic charge quantum is independent of $\hbar$. The magnetic flux tube that detects Z_N electric charge is a classical object, and the action of its world sheet is independent of $\hbar$. Hence, the effects of virtual strings are nonperturbative in $\hbar$. In contrast, the electric flux tube that detects Z_N magnetic charge is a quantum–mechanical object with tension of order $\Lambda^2/\hbar$ and thickness of order $\hbar/\Lambda$; loops of string with radius comparable to the string thickness have world sheet "action" of order $\hbar$, and occur copiously as quantum fluctuations. This behavior can be reconciled with weak–coupling behavior of the quantum fields near the event horizon because of asymptotic freedom.

In spite of this profound formal difference between Z_N electric and magnetic hair in the small–$\hbar$ limit, their qualitative physical consequences are not so drastically different. We have already seen that, although a black hole with Z_N electric charge has no classical electric field outside the event horizon, a nonzero electric field *is* generated by quantum effects. On the other hand, in the case of screened magnetic charge in the thin–string limit, one might wish to use an effective Lagrangian that incorporates the screening from the start. In that effective theory, classical no–hair theorems would apply to screened magnetic charge.

As a black hole with fixed Z_N magnetic charge evaporates, it cools, and the Hawking process eventually shuts down. The mass M of a zero–temperature hole is given by[11]

$$M^2/\hbar = \left(\frac{4\pi}{\hbar e^2}\right) \frac{n(N-n)}{2N} , \tag{8}$$

where $n = 1, 2, \ldots, N-1$ is the Z_N charge, and e is the gauge coupling (renormalized at a distance scale of order $R = 2M$). For a theory that is weakly coupled at the event horizon, M is much larger than the Planck mass, so that semiclassical methods are reliable.

These zero–temperature black holes do not emit Hawking radiation, but they *can* decay to elementary monopoles, if the elementary monopoles are light enough. On the other hand, if the elementary monopoles are sufficiently heavy, then the elementary monopoles *are* ($n = 1$) black holes.[16] In that event, the extreme magnetically charged black holes, for all values of n, are kinematically forbidden to decay into lighter objects with the same total magnetic charge. They are absolutely stable black hole remnants, stabilized by the Z_N magnetic charge that they carry.

Axion Hair: Thus far we have focused our attention on the quantum hair associated with a discrete gauge symmetry. What if the discrete symmetry is a *global* symmetry rather than a gauge symmetry?

It is generally believed that *continuous* global symmetries are violated (or transcended) by black holes. Particles that carry globally conserved charges can fall into a black hole; if the black hole has no internal quantum numbers, this process flouts the conservation law, and so the global symmetry loses its power. In particular, if the black hole eventually evaporates completely and disappears, it has no reason to disgorge the same amount of charge as it has swallowed.

It was emphasized in Ref. 6 that the situation could be different if a continuous global symmetry is broken to a discrete subgroup (*e.g.* global $U(1)$ broken to Z_N). Such a symmetry breaking pattern admits a *global* cosmic string, and one is tempted to argue that the string has an Aharonov–Bohm interaction with an object that carries the globally conserved Z_N charge. Formally, this scheme is just the $e \to 0$ limit of the Z_N gauge model that we have already discussed, and no e appears in the Aharonov–Bohm phase $\exp(i2\pi/N)$ acquired by a unit charge that circumnavigates the minimal string. However, this limit may be singular—see below.

If the global Z_N charge can indeed be detected at infinite range by means of the Aharonov–Bohm effect, then black holes should be able to carry global Z_N hair. It was noted in Ref. 6 that the Goldstone boson field associated with the symmetry breakdown is related by a duality transformation to an antisymmetric tensor field B, and that black hole solutions can be constructed with nonzero B field and hence nonzero global charge, or "axion" charge. (The axion charge inside a closed two–surface is the integral of B over the surface.) The advantage of this dual formulation is that it allows us to discuss axion hair in the language of classical field theory, and to exhibit a black hole with axion hair as a solution to classical field equations. Nonetheless, axion hair is really a variety of quantum hair, for the B field of a black hole can be detected only by means of the Aharonov– Bohm effect. In fact, the same duality transformation can be performed in a theory with a gauged Z_N symmetry, enabling us to exhibit a black hole with gauge quantum hair as a classical solution, too.[11,17]

Let us consider how global Z_N hair influences black hole thermodynamics. The partition function of a black hole with specified global charge is given by a formula like eq. (5), but with Q/Ne replaced by the charge of the black hole in units of the charge of the condensate, and where Z_k is the path integral in the sector with *global* vorticity k; *i.e.* such that the phase of the condensate rotates by $2\pi k$ as τ varies from 0 to β. Now, however, there are no vortex configurations of finite action in the infinite volume limit; in fact, the action of a vortex diverges like R^3 with the radius R. Thus, the $k \neq 0$ contribution to the partition function is snuffed out, and with it the charge dependence. This result appears at first sight paradoxical, since one can argue on general grounds that *if* the axion charge (modulo N) is a well-defined conserved quantity, then the charge on a black hole *must* alter the evaporation process.[6,18]

Since virtual global strings are impotent (in the context of black hole physics), one is led to question the virility of real ones. Indeed, there are important distinctions, both formal and physical, between gauge and global discrete symmetries; these distinctions suggest that only the former give rise to meaningful quantum hair.

The important formal difference emerges most clearly in the dual formulation mentioned above, where the charge Q inside a closed surface is represented as the integral $Q = \int B_{\mu\nu}\, d\sigma^{\mu\nu}$. Whether the symmetry is global or local, the antisymmetric tensor field B appears in the classical field equations only in the combination $H_{\mu\nu\rho} \equiv \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$ $(H = dB)$. But when the symmetry is gauged, the coupling of the Goldstone boson to the gauge field assumes a form[11]

$$L_{\rm dual} = -\frac{1}{4}\, \epsilon^{\mu\nu\rho\sigma}\, B_{\mu\nu}\, F_{\rho\sigma} \tag{9}$$

that cannot be expressed in terms of the field strengths H and F. In this case, then, a B field that is closed ($H = 0$) but not exact ($B \neq d\Lambda$) can be observable. (Only the cohomology class of a closed B is observable; an exact B does not contribute to the action, in the absence of magnetic monopoles, because $dF = 0$.) When the symmetry is global, however, the action is a functional of H, and a closed B has no physical effects.

Now on the Schwarzschild background (which has topology $S^2 \times R^2$), there is a closed two–form that is not exact—it is $\epsilon/4\pi r^2$, where ϵ is the induced volume form on the two– sphere. Since only H (not B) appears in the classical field equations, one can construct a black hole solution with $B = Q\epsilon/4\pi r^2$. This solution is a black hole with charge Q, and the solution exists irrespective of whether the symmetry is global or local. When the symmetry is gauged, the coupling in eq. (9) induces an Aharonov–Bohm interaction of the black hole with cosmic strings. In the Euclidean path integral, this coupling gives an imaginary contribution to the action, proportional to the vorticity, that reproduces the phase factor in eq. (5). But in the case of global symmetry, the charge Q exerts no influence on quantum field theory outside the event horizon, for the closed part of B decouples from the action.

In more physical terms, a crucial distinction between the cases of gauge and global discrete symmetry is that there is no mass gap in the global case. In classical field theory, states of arbitrarily low energy can be constructed (in an infinite volume) with any given value of the charge. Thus the thought experiment, wherein one slowly envelops an object with a large string world sheet to measure its charge, involves a subtlety—in a system with massless charged particles, fractional charge can leak out of a large detector with finite energy resolution. It may be, then, that the charge spectrum of the theory is ill-defined even if, formally, a discrete subgroup of the the global symmetry group remains manifest. If so, there need be no particles with Z_N charge that can be detected at infinite range, and hence no means of depositing such charge on a black hole.

We have attempted to characterize the dynamical effects of axion hair using an effective field theory formalism, and have found no effects. It is conceivable that the conclusion would be different if we included a coupling of axion charge to *fundamental* strings, as contemplated in Ref. 6. However, in a realistic string theory, the fundamental string is actually the boundary of an axion domain wall. This wall decays by nucleation of string loops, and so the Aharonov–Bohm interaction of charge with the string can *not* be used to measure charge at arbitrarily long range. In any event, a proper investigation of quantum hair on black holes in the context of string theory would require a more sophisticated analysis than that described in this paper.

Conclusion: If a gauge theory is in a Higgs phase, electric fields are screened, and a black hole cannot carry classical electric hair. But *quantum* hair associated with a discrete gauge symmetry can nevertheless reside on a black hole. This quantum hair generates an electric field outside the event horizon that is nonperturbative in $\hbar$, and has calculable effects on the thermodynamic behavior of the hole. If a gauge theory is in a confining phase, magnetic fields are screened. Classical magnetic hair can nevertheless reside on a black hole, and can support a stable black hole remnant. However, the proposed quantum (axion) hair associated with a discrete *global* symmetry has no dynamical effect that we can discern, which leads us to question its existence.

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