Physics 12C, Spring 2008
Midterm Examination

Due on Thursday, May 8

Instructions — Read this before you start.
This is a 4-hour closed book examination. You are permitted to use your own hand-written notes or lecture notes posted on the class web site. You are, however, not allowed to look at your graded homework sheets or homework solutions sets while working on the examination. You can use a pocket calculator (though I do not think you would need it), but you are not allowed to use a desktop or a laptop computer. Please try to answer all questions, and show your work to maximize the possibility of partial credit.

Fundamental Constants:

Speed of Light $c = 3 \times 10^8 \text{m/s}$, Planck Constant $\hbar = 1 \times 10^{-34} \text{Js}$
Boltzmann Constant $k_B = 1.4 \times 10^{-23} \text{J/K}$, Avogadro Number $N_A = 6.0 \times 10^{23}$

[1] This is a variation on the "drunk on a street with lamp posts" problem. The lamp posts are numbered with consecutive integers, and on every time step the drunk moves either one lamp post to the right or one lamp post to the left. This time, however, the street is sloped, so there's a preferred direction for the drunk - downhill. Suppose the drunk starts at lamp post 0 and has a constant probability p of moving to the right (in the direction of increasing lamp post numbers).

a) What is the probability that the drunk is at lamp post n after N time steps?

b) What is the expected location of the drunk after N steps, $\langle n \rangle$, and its varianlce, Δn?

c) Check that your result in part b) is reasonable by computing $\langle n \rangle$ and Δn for $p = 0.5$ and $p = 1$ and comparing with expectations.

d) What is the ratio $\langle n \rangle / \Delta n$ as a function of N? We can interpret this as the signal-to-noise ratio for this problem.

[2] We will model the unfolding of a protein molecule under the action of a force in 1 dimension. Suppose the molecule consists of N segments of length d each, with total length $L = Nd$. Each segment can be represented by a vector of length d that points to either to the right or to the left, with each subsequent segment starting at the endpoint of the previous segment. One end of the molecule is tied down, while the other end is acted on by a force F to the right. A segment that points to the right (along the force direction) has energy $-Fd$, while a segment that points to the left (against the force, energetically unfavorable) has energy $+Fd$. All of this occurs in a bath of temperature $T = \tau / k_B$.

a) Compute the partition function Z.

b) Compute the free energy, entropy of the molecule, and energy of the molecule.

c) Compute the expected length of the protein molecule (the separation between its two ends) in terms of F, L, N and T.

d) Approximate the length in both the high- and low- temperature limits.

[3] Consider a D-dimensional hypercube blackbody cavity. What is the energy density (up to a multiplicative constant) as a function of temperature? It is not necessary to derive the multiplicative factor. Check your result for $D = 3$ to make sure it gives the right dependence.

[4] The rotational energy levels of a diatomic molecule is given by $E_j = j(j+1)\epsilon_0$ for some $\epsilon_0 > 0$, where the quantum number $j = 0, 1, 2, 3, \ldots$. There are $(2j+1)$ states at the energy level E_j.

a) Find the partition function $Z_{rot}(T)$ for the rotational modes of one molecule.

b) Evaluate $Z_{rot}(T)$ when $kT \gg \epsilon_0$ by converting the sum to an integral. Also evaluate $Z_{rot}(T)$ when $kT \ll \epsilon_0$.

c) Find expressions for the energy E and the heat capacity C_V at constant volume for $kT \gg \epsilon_0$ and $kT \ll \epsilon_0$.

[5]

a) Intergalactic space is believed to be occupied by hydrogen atoms in a concentration ~ 1 atom m^{-3}. The space is also occupied by the Cosmic Microwave Background Radiation at 2.9 K. Compute the ratio of the heat capacity of matter to that of radiation.

b) Consider a dielectric solid with a Debye temperature equal to 100K with 10^{22} atoms cm^{-3}. Estimate the temperature at which the photon contribution to the heat capacity would be equal to the phonon contribution evaluated at 1K.