Physics 12, Spring 2008
Problem Set 1

Date assigned: April 3 **Due on April 10 at 5:00 pm**

Homework sets should be put in the Physics 12 lock box outside 269 Lauritsen.

Reading Assignment: Chapter 1 of Kittel-Kroemer

[1] (Molecules in a Box)

Consider N distinguishable molecules with identical properties. Assume there are no interactions between molecules. Put them in a box, which is partitioned into two parts, A and B, of the same size and shape. Molecules can freely move between A and B. Assume that the probability that a given molecule is in part A is 1/2.

(i) Show that the probability $P(N_A)$ of N_A molecules in the part A is

$$P(N_A) = \frac{{}_N C_{N_A}}{2^N}$$

(ii) Show that this probability distribution reduces to the Gaussian distribution

$$P_{Gaussian}(N_A) = \sqrt{\frac{2}{\pi N}} \exp\left[-\frac{2(N_A - \frac{N}{2})^2}{N}\right]$$

in the limit when $|N_A - \frac{N}{2}| \ll N$.

(iii) Now, consider the case when the volumes of A and B are different and that the probability of finding a particular molecule on the side A is $p = v/V$ where v is the volume of A and V is the total volume of the box. The total number of the molecules on the box is N. Compute the probability that the number of molecules on the side A is N_A.

(iv) What is the large N limit of the probability distribution when we assume that the probability p is kept finite as one takes the limit $N \to \infty$? Compute the mean $\langle N_A \rangle$ and the standard deviation $\sqrt{\langle (N_A - \langle N_A \rangle)^2 \rangle}$ of the distribution. Here, for a function $f(N_A)$, the symbol $\langle f(N_A) \rangle$ means its average.

[2] (Random Walks)

(i) A drunk starts out from a lamppost in the middle of a street, taking steps of equal length either to the right or to the left with equal probability. What is the probability that the man will again be at the lamppost after taking N steps?

Hint: This problem is closely related to the problem [1] in the above.

(ii) Two drunks start out together at the lamppost, each having equal probability of making a step to the left or right. Find the probability that they meet again after N steps. It is to be understood that they make their steps simultaneously.

Hint: It may be useful to consider their relative motion.

[3] (Large N, Small p)
Let us continue to study the probability distribution discussed in the above problem [1](iii). Unlike the case of [1](iv) where we assumed p is kept finite, here we consider the case when the probability p is very small ($p \ll 1$). Since $p = v/V$, this means that the volume v of A is very small compared to the total volume V of the volume.

For definiteness, let us set $p = \frac{\lambda}{N}$ and take $N \to \infty$ while keeping λ finite.

(i) Let us obtain the large N – small p limit of the probability distribution of the problem [1](iii) in the following steps.
(a) By using the Taylor expansion $\log(1-p) = -p + O(p^2)$, show that $(1-p)^N$ approaches to $e^{-\lambda}$ in this limit.
(b) Show that $N!/(N-N_A)!$ is approximately N^{N_A} when $N_A \ll N$.
(c) By using these result, show that the probability distribution of [1](iii) reduces to

$$P_{Poisson}(N_A) = \frac{\lambda^{N_A}}{N_A!} e^{-\lambda}.$$

This is called the Poisson distribution. The Poisson distribution appears when a probability of each individual incident is small (such as here when the probability of finding a particular billiard ball in the side A is small).

(ii) Show that the Poisson distribution is correctly normalized, namely

$$\sum_{n=0}^{\infty} P_{Poisson}(n) = 1.$$

Compute the mean and the standard deviation of the distribution.

(iii) Assume that typographical errors committed by a typesetter occur completely at random. Suppose a book of 600 pages contain 600 such errors. Calculate the probability
(a) that a page contains no errors
(b) that a page contains at least 3 errors