Physics 12, Spring 2008
Problem Set 7

Date assigned: May 22  Due on May 29 at 5:00 pm

Homework sets should be put in the Physics 12 lock box outside 269 Lauritsen.

Reading Assignment: Chapters 8 of Kittel-Kroemer.

Do problems 3, 6, 7, and 9 in Chapter 8.

In addition, do the following problems:

[1] Consider the Fermi-Dirac distribution,

\[ f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon - \mu}{kT}} + 1}. \]

The density of states is given by \( D(\varepsilon) d\varepsilon \). Consider a physical observable \( \psi(\varepsilon) \) and its expectation value \( I \) given by

\[ I = \int_0^{\infty} \psi(\varepsilon) D(\varepsilon) f(\varepsilon) d\varepsilon. \]

We should adjust the chemical potential \( \mu \) so that the number of particles is \( N \). Suppose the temperature \( \tau \) is much below the Fermi energy \( \varepsilon_F = \mu(N, \tau = 0) \).

Show

\[ \left( \frac{\partial I}{\partial \tau} \right)_\mu = \frac{\pi^2}{3} \tau \frac{d}{d\varepsilon} (\psi(\varepsilon) D(\varepsilon))_{\varepsilon=\varepsilon_F} + O(\tau^3), \]

\[ \left( \frac{\partial I}{\partial \mu} \right)_\tau = \psi(\varepsilon_F) D(\varepsilon_F) + O(\tau^2), \]

\[ \left( \frac{\partial I}{\partial \tau} \right)_N = \frac{\pi^2}{3} \tau \frac{d}{d\varepsilon} (\varepsilon_F) D(\varepsilon_F) + O(\tau^3). \]

[2] Suppose there is some material whose internal energy (when expressed as a function of the volume and the temperature) is independent of the volume. Show that:

(a) the heat capacity \( C_V \) depends only on the temperature \( \tau \),

(b) the volume depends only on the ratio \( P/\tau \) of the pressure and the temperature, and

(c) the difference \( (C_P - C_V) \) depends only on \( P/\tau \).