Problem 1

a. Argue that if the degeneracy does not disappear after diagonalizing the perturbing Hamiltonian $H^{(1)}$ on the space of degenerate states $|n_j^{(0)}\rangle$ then one must diagonalize the matrix $W_{ij}$ defined by

$$W_{ij} = \langle n_i^{(0)} | H^{(1)} | n_j^{(0)} \rangle + \sum_m \frac{\langle n_i^{(0)} | H^{(1)} | m^{(0)} \rangle \langle m^{(0)} | H^{(1)} | n_j^{(0)} \rangle}{E_0^n - E_0^m}$$

to find the energy levels at second order in perturbation theory.

b. A spin $1/2$ particle of mass $m$ moves in a spherical harmonic oscillator potential $V = \frac{m}{2} \omega^2 r^2$ and is subject to an interaction $\lambda \mathbf{S} \cdot \mathbf{r}$ where $\mathbf{S}$ is the spin-operator. The Hamiltonian is thus $H = H^{(0)} + H^{(1)}$ where

$$H^{(0)} = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 r^2, \quad H^{(1)} = \lambda \mathbf{S} \cdot \mathbf{r}$$

Calculate the ground state energy through second order in perturbation theory.

Problem 2 (Exercise 17.34 part (3))

a. For the hydrogen atom calculate

$$\langle n, l, m | 1/r^3 | n, l, m \rangle$$

(Hint: Use the trick described in the Shankar.)

Problem 3

The spin-dependent Hamiltonian of an electron-positron system in the presence of uniform magnetic field pointing along the $z-$direction may be written as:

$$H = A \vec{S}_1 \cdot \vec{S}_2 + \frac{eB}{mc} (S_{1z} - S_{2z})$$

where $\vec{S}_1$ is the spin operator for the electron (with mass $m$ and charge $-e$), while $\vec{S}_2$ is the spin operator for the positron (with mass $m$ and charge $+e$).

a. Find the energy eigenstates and the corresponding energy eigenvalues when $B = 0$.

b. Now consider $B$ to be very small, and treat the second term in $H$ using time-independent degenerate perturbation theory. Find the leading order corrections to the energies.

c. We now attempt to cause transitions (via stimulated emission and absorption) between the two $m = 0$ total spin states by introducing an oscillating magnetic field of the right frequency. Should we orient the magnetic field along the $z-$axis, or along the $x-$ (or $y-$) axis? Justify your answer.

d. Calculate the energy eigenvectors to first order.