Unification in physics

Often resolution of inconsistency \(\Rightarrow\) unification

- elegant
- not optional
- new predictions

Electromagnetism

First electrostatics \((p = \text{const. in } t)\)

magnetostatics \((j = \text{const. in } t)\)

Add dynamics \(\Rightarrow\) \(E + M\) mix

Faraday: \[
\oint E \cdot dl = -\frac{1}{c} \frac{d}{dt} \oint B \cdot dA
\]

(Can't have changing \(B\) without an \(E\).

Maxwell: Noticed inconsistency of Ampère's law:

New term \(\Rightarrow\) Maxwell's equations,

complete unification of \(E + M\).

Predicts EM waves.

What was inconsistency?

Ampère's law:

\[
\oint_S \mathbf{B} \cdot d\mathbf{l} = \frac{1}{c} \int_S \mathbf{J} \cdot d\mathbf{A}
\]
Consider a changing capacitor:

\[ \mathbf{J} \cdot \mathbf{dA} = I, \]

\[ \mathbf{E} \cdot \mathbf{dA} = 0. \]

Resolution: add displacement current \( \mathbf{J}_D = \mathbf{\partial E}/\partial t \).

\[ \int_{S_2} \mathbf{E} \cdot d\mathbf{A} = Q \quad \text{(Gauss's Law)} \quad \Rightarrow \quad \int_{S_2} \mathbf{\partial E}/\partial t \cdot d\mathbf{A} = \dot{Q} = I. \]

Next two unifications:

- EM Lorentz invariant, but Newtonian Mechanics Galilean invariant.

**Special Relativity (SR):** space + time unification.

- UV catastrophe (black bodies should radiate infinitely in UV), atomic instability (\( e^- \) should radiate and spiral into nucleus), photoelectric effect, ...

**Quantum Mechanics (QM):** wave + particle unification.

In the Standard Model of Particle Physics (SM), all three forces are compatible with SR and QM:

1. **Quantum Electrodynamics (QED):** quantum EM

   photon (massless)
2. **Weak Interactions**: \( \beta \)-decay, processes w. neutrinos (\( \nu \))

\( W^+, W^-, Z \) (mass \( \approx 10^2 \text{ GeV}/c^2 \))

3. **Strong Interactions**: bind quarks into \( p, n, \text{nuclei} \)

8 gluons (massless)

The theory of the strong interactions is also known as Quantum Chromodynamics (QCD).

In the 1960s, consistency of QED + weak gave

1+2. **Electroweak Interactions (EW)**:

At energies \( E \approx 10^2 \text{ GeV} \), EW unbroken.

At \( E \approx 10^2 \text{ GeV} \), choice of groundstate of Higgs boson

(i) splits EW to QED + weak

(ii) gives \( W^+, W^-, Z \) masses and leaves photon massless.

This EW unification is part of the SM.

Possibility beyond the SM: EW + QCD unify to one Grand Unified Theory (GUT) at high energy \( E \approx 10^{16} \text{ GeV} \).

Last force, not described by SM is gravity.

4. **SR incompatible w.**

Newtonian gravity \( \Rightarrow \) **General Relativity**

GR is a non QM theory in which spacetime becomes dynamical.
What's wrong with classical GR + QM Standard Model?

Here's an argument from Wald, General Relativity, Sec 14.1:

Suppose \( \text{Prob} = \frac{1}{2} \) QM matter is in region \( O_1 \),
\( \text{Prob} = \frac{1}{2} \) QM matter is in region \( O_2 \),

and suppose that the gravitational field is sourced by \( \langle \text{matter} \rangle \).
(Roughly, \( \Phi_{\text{Grav}} = \sum_i G_{\text{Nm}} |i\rangle \langle i| \langle \psi | \frac{1}{\sqrt{i}} | \psi \rangle \), generalized to GR.)

Then, the gravitational field is the same as that due to classical matter, half in region \( O_1 \) and half in region \( O_2 \).

Now measure \( |\psi\rangle \rightarrow \text{eigenstate} \langle 0_1 | \text{or} \langle 0_2 | \).
The classical gravitational field changes acausally.

More satisfactory to say gravitational field is QM, \( \Psi \).

\( \text{Prob} = \frac{1}{2} \) field due to matter in \( O_1 \),
\( \text{Prob} = \frac{1}{2} \) field due to matter in \( O_2 \).

Why isn't this a big practical problem?

Usually, can neglect either gravity or QM/Standard Model.

Astronomical Scales: macroscopic objects
large particle numbers
no significant QM coherence
(i.e., not all in phase) \( \Rightarrow \) neglect QM,

masses add, charges cancel \( \Rightarrow \) neglect SM forces.
Atomic scales: \[ \frac{G_N m_e}{r^2} = 10^{-42} \frac{e^2}{4\pi e^2} \Rightarrow \text{neglect gravity.} \]

Accelerators: \[ m_e \rightarrow E/c^2 = \gamma m_e \]
\[ e \rightarrow e'(E) \]

But gravity still \( \ll \) SM forces \( \Rightarrow \) neglect gravity.

When IS quantum gravity important?

- In SR + QM, a particle of mass \( m \) in its rest frame \( \langle \vec{p} \rangle = 0 \) can only be localized within

\[ \Delta x \approx \frac{\hbar}{2mc} = \lambda_c \quad \text{(Compton wavelength).} \]

\[ \Delta x \approx \frac{\hbar}{\Delta p}, \quad \text{but for } \Delta p \approx mc, \quad \text{have pair production.} \]

- In GR, can't make \( m \) too big or get a black hole:

\[ R_s = \frac{2G_N m}{c^2} \quad \text{(Schwarzschild radius).} \]

\[ \text{event horizon: nothing gets out from } \quad \frac{R}{R_s} \quad \text{can't see inside.} \]

- So, if particle not a black hole, \( R_s \ll 1 \Leftrightarrow \frac{G_N m}{c^2} \ll \frac{\hbar}{mc} \]

\[ \Leftrightarrow m \ll \sqrt{\frac{\hbar c}{G_N}} \equiv m_p \quad \text{(Planck mass)} \]

\[ \Delta x \approx \frac{\hbar}{mc} \approx \sqrt{\frac{G_N \hbar}{c^5}} \equiv l_p \quad \text{(Planck length)} \]

This is the smallest meaningful length. At length scales \( \Delta x \ll l_p \), expect fuzzy quantum geometry, and likewise for timescales \( \Delta t \ll t_p \), where \( t_p = l_p/c = \sqrt{G_N \hbar / c^5} \quad \text{(Planck time)}. \)