Physics 135c, String Theory

Problem Set 4
Spring 2005

Reading: Please read Zwiebach Chapter 8 and Sections 9.1–9.3 of Chapter 9.

Problems: Please do Problems 1 and 2. Then pick any two of Problems 3–7, or do more than two for extra credit.

1. Open string endpoints move at the speed of light. Zwiebach Problem 6.5.


Note that \( \vec{P}(t, \sigma) \) in this problem is the same as the spatial part of the four-vector \( P^\mu_\tau \) defined in Eq. (6.49) of the text. The definition of the Hamiltonian density in terms of the Lagrangian density is

\[
\mathcal{H} = \vec{P} \cdot \vec{v} - \mathcal{L}, \quad \text{where} \quad \vec{v} = \partial_t \vec{X}.
\]

3. Stretched string and nonrelativistic limit. Zwiebach Problem 6.1


5. Planar motion for open string with fixed endpoints. Zwiebach Problem 6.3.


7. Newtonian limit of point particle motion in curved space.

On Problem Set 2, you showed that in the weak field nonrelativistic limit, the spacetime interval takes the form

\[
-\text{d}s^2 = g_{\mu\nu}(x)\text{d}x^\mu\text{d}x^\nu = (\eta_{\mu\nu} + h_{\mu\nu}(x))\text{d}x^\mu\text{d}x^\nu \\
\simeq -\left(1 + \frac{2\Phi_g}{c^2}\right)(\text{d}x^0)^2 + \left(1 - \frac{2\Phi_g}{c^2}\right)((\text{d}x^1)^2 + (\text{d}x^2)^2 + (\text{d}x^3)^2).
\]

Here, \( \Phi_g \) is the Newtonian potential, with \( \Phi_g/c^2 \ll 1 \).

If you chose to do Problem 7 on last week’s problem set, you also showed that starting from the spacetime interval \( \text{d}s^2 = -g_{\mu\nu}(x)\text{d}x^\mu\text{d}x^\nu \), the equation of motion that follows from the point particle action

\[
S = -mc \int \text{d}s
\]

is

\[
\frac{\text{d}x^\mu}{\text{d}s^2} + \Gamma^\mu_{\rho\sigma} \frac{\text{d}x^\rho}{\text{d}s} \frac{\text{d}x^\sigma}{\text{d}s},
\]

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where the Christoffel coefficient $\Gamma^\mu_{\rho\sigma}$ is defined by

$$\Gamma^\mu_{\rho\sigma} = \frac{1}{2} g^{\mu\lambda} \left( \frac{\partial g_{\lambda\rho}}{\partial x^\sigma} + \frac{\partial g_{\lambda\sigma}}{\partial x^\rho} - \frac{\partial g_{\rho\sigma}}{\partial x^\lambda} \right).$$

This equation of motion is called the geodesic equation. It describes the straightest possible paths or geodesics in curved spacetime. (Don’t worry if you didn’t do this problem last week. The result is all that is needed here.)

Working to leading order in $\Phi_g/c$ and $v/c$, where $v = dx/dt$, show that the geodesic equation reduces to the Newtonian equation of motion:

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial}{\partial x^i} \Phi_g.$$