Physics 135c, String Theory

Problem Set 5

Spring 2005

Reading: Please read Zwiebach Sections 9.4–9.5 and all of Chapter 10.

Problems: Please do any two of Problems 1–3 and one of Problems 4–5. As usual, additional problems will count for extra credit.

1. Energy density and effective tension. On last week’s problem set, you computed the Hamiltonian for a relativistic string in static gauge and provided a simple physical interpretation for the result. This was used in Zwiebach, Sec. 7.2 to confirm that

\[ \mathcal{E}ds = T_0 ds / \sqrt{1 - v^2_\perp c^2} \]

indeed gives the energy of a small length \( ds \) of string. On the other hand, the effective tension of Eq. (7.18),

\[ T_{\text{eff}} = T_0 \sqrt{1 - v^2_\perp c^2}, \]

was left uninterpreted.

(a) Provide a physical interpretation for the effective tension. (Hint: In the last part of Problem 1 on Problem Set 3, you explored the Lorentz transformation of force in a restricted context that also suffices here.)

(b) Zwiebach Problem 7.2(a). Energy density \( \mathcal{E}(s) \) for the rigidly rotating string.

2. Zwiebach Problem 7.3. Time evolution of an initially static closed relativistic string.

3. Zwiebach Problem 7.7. Three dimensional motion of closed strings and cusps. This is a new problem that will appear in the next edition of the textbook. It is available through the supplementary materials link on the course web page.

In this problem, you will show that “instantaneous cusps moving with the speed of light appear generically in the free motion of closed strings in three dimensions.” This is a result of practical importance. One possible observable consequence of string theory is that very small strings produced in the early universe could have grown with the expansion of the universe to cosmic strings of astronomical size. Each time these cosmic strings form a cusp, they are expected to emit a characteristic spectrum of gravitational radiation that could in principle be detected by LIGO, the Laser Interferometer Gravitational-Wave Observatory.

4. Zwiebach Problem 8.3. Lorentz charges for the relativistic point particle.
5. **Zwiebach Problem 8.7.** Generalizing the construction of conserved currents.

In class, we discussed the conserved currents associated with translations in spacetime. In this problem, you will explore a more general formalism that can be used when $\delta L$ is a total derivative rather than zero. This is needed, for example, to discuss translations in parameter space.